

# *Arithmetic for Parents*

*A Book for Grownups  
about  
Children's Mathematics*

by  
Ron Aharoni

Translated from Hebrew  
by Danna Reisner



Sumizdat



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# Foreword

Most adults have long buried their memories of studying mathematics. All they really want is to forget the trauma. They accept their past incomprehension as a tolerable, albeit painful, fact. “You don’t really need to know mathematics,” they console themselves. Until one day the need does arise and the old anxieties resurface. This happens when their child begins dealing with the same experiences.

Many people would like to help their children with their arithmetic studies, but are too afraid to return to a subject that was so painful for them as children. However, they forget that they have acquired many new tools since their schooldays. The adult has more forbearance, abstraction skills, the capability to deal with complex sentences, and the patience to wait until the whole picture emerges. All these skills can be used to deal more easily and quickly with the principles of elementary school mathematics.

The purpose of this book is to provide the guidance required to do so. It offers assistance to the parent who wishes to be an active participant in his or her child’s arithmetic studies. As a matter of fact, that is how the book was born. The parents at my son’s school asked for written instructions, so that they could help their children with mathematics. What began as modest notes, slowly evolved and eventually reached the proportions of a book.

Yet books, like ideas, have a life of their own. Sometimes they lead their writer no less than he leads them. And thus it happened that the book gradually took on a different form. One of the insights I came by while teaching in elementary school, is that elementary

mathematics isn't simple at all. It has depth and beauty. This message slowly found its way into the book, and gave it an additional direction: a description of the beauty of elementary mathematics, and, consequently, mathematics in general. Thus, my original target audience expanded to include the reader who wishes to return to his childhood mathematics, from a different angle. For this group of readers, the book provides a second chance. Those who learned how to multiply fractions or how to perform long division, but never understood why it was done exactly so, are invited to take a look from a new, mature perspective.

The book is also aimed at a third audience, no less important for me: teachers and educators. To these the book sends a clear message: proper teaching of mathematics depends more on an understanding of the mathematical principles than on educational tricks. It requires familiarity with the way the fine mathematical layers lie one upon the other. Furthermore, it is best done by direct, concrete experience with the concepts, with no intermediaries.

The first part of the book, *Elements*, describes the fundamentals of elementary mathematics. Namely, it deals with the questions of what is mathematics, what is the material that should be taught in elementary school, and also what makes mathematics beautiful and in what ways is mathematics similar to the arts.

The parent who wishes to help his or her child must be familiar, beyond mathematics, with basic teaching principles, and to these I have dedicated the second part of this book. It includes the principal rules of progressing from the concrete to the abstract. In addition, parents must also be familiar with the educational trends that determine the educational character of their child's school. For this reason, the book includes an appendix describing, in a nutshell, the major developments in mathematical education over the last fifty years.

The third part of the book is devoted to the nitty-gritty details of elementary mathematics. Namely, it unfolds the material taught in school, stage by stage. Although geometry constitutes between 10% and 20% of the usual curriculum, I decided to renounce it, and stick to arithmetic — that is, the study of the properties of numbers. Besides the centrality of arithmetic in the curriculum, the reason for this choice is that arithmetic is a uniform, polished body of knowledge, not unlike a fine diamond, and it is worthy of a book dedicated to it alone.

## *First Lesson*

*You never get a second chance  
to make a good first impression.*

An American saying

In education, as in life, first impressions are important. The first time a subject is introduced will determine, to a great extent, the future attitude towards it — will it remain a pleasant memory, or a painful one? Will it arouse a feeling of “I understand,” or of “this is difficult”?

For this reason, chapters titled *First Lesson* are interwoven throughout the book. They provide suggestions of possible ways to present subjects for the first time. There is never a single method, but it is always helpful to have a few options. The ideas are presented as pointers for teachers, but parents may also find them useful.

